

On some links between quantum physics and gravitation

Aleksey V. Ilyin*

Moscow Institute of Physics and Technology

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It is widely believed that quantum gravity effects are negligible in a conventional laboratory experiment because quantum gravity should play its role only at a distance of about Planck's length ($\sim 10^{-33}$ cm). Sometimes that is not the case as shown in this article. We discuss two new ideas about quantum physics connections with gravity. First, the Hong-Ou-Mandel effect relation to quantum gravity is examined. Second, it is shown that the very existence of gravitons is a consequence of quantum statistics. Moreover, since the Bose-Einstein statistics is a special case of Compound Poisson Distribution, it predicts the existence of an infinite family of high-spin massless particles.

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I. INTRODUCTION

It is a common knowledge that gravity plays no role in conventional quantum phenomena because the gravitational interaction is negligible if compared to electromagnetic or strong interaction. The role of gravity in quantum mechanics is usually illustrated by the following example [1]: radiative decay of an excited state of an atom due to electromagnetic interaction takes about 10^{-10} sec, whereas the same decay due to gravitational interaction must go on for about 10^{30} years, which is much longer than the age of the Universe.

Such an insignificant role of gravity in atomic processes, however, does not prevent it from determining the motion of celestial bodies and playing a prominent role in people's daily life. Gravity acts in the first place where all other forces are compensated or absent. But similar situation is also possible, as shown below, in some quantum processes. This requires that no other interactions, such as electromagnetic, interfere with quantum gravity effects.

In Section II, the Hong-Ou-Mandel experiment [2] is analyzed for quantum gravity effects. We start from a short description of the HOM-effect, then we prove that coalescent photons observed in the experiment do not obey the Maxwell's equations for electromagnetic field, and finally we discuss what equations may be used to describe coalescent photons. Strong reasons are presented in support of conclusion that coalescent photons obey the Einstein's equations of general relativity.

Another example of quantum physics connection to gravity is considered in Section III, which shows that quantum statistics necessarily predicts the existence of gravitons and an infinite family of high-spin massless particles provided there is a spin-one massless particle (photon). This conclusion follows from the fact that the Bose-Einstein (BE) statistics coincides with negative binomial distribution, which, as shown in [3], is a special case of Compound Poisson Distribution. Wherefrom it follows that the BE statistics describes a stream of composite

random events, each composite event consisting of random number of elementary events. In quantum optics, an elementary event may only be a registration of single photon, so that a composite event is a registration of photon cluster, i.e. a high-spin massless particle.

Main results of this work are discussed in Section IV.

II. HOM-EFFECT

In the famous experiment by Hong, Ou and Mandel [2] two identical photons a and b impinge on two different input ports of a symmetric beam splitter (Figure 1). After interacting with the beam splitter both photons are found in a single output mode - either in mode c or in mode d . Such photon pairs in a single mode were called the coalescent photons [4].

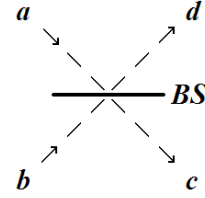


Figure 1. Hong-Ou-Mandel effect: two identical photons a and b , upon impinging on a beam splitter (BS), are found either in the output mode c or in the output mode d . Therefore, such photons are deemed coalescent.

Two coalescent photons show a remarkable behavior: if they are allowed to fall on a second beam splitter (not shown) then they are either together transmitted or together reflected at the beam splitter [5]. In other words, coalescent photons behave as if they were a single quantum object that is never separated into two different photons when interacting with a semi-transparent mirror. Therefore, it is convenient to call such an object a two-photon cluster [6].

Apart from two coalescent photons, three and even more coalescent photons were observed in different experiments [7–10]. For that reason, N coalescent photons will be termed here as the N -photon cluster.

* a.v.ilyin@mail.mipt.ru

A. Photon clusters do not obey Maxwell's equations

It is important that N -photon cluster field does not obey the Maxwell's equations for electromagnetic field. Indeed, the momentum of N -photon cluster is N times the momentum of a single photon. According to de Broglie's formula $\lambda = \frac{h}{p}$, the wavelength of a particle is determined by its momentum. Therefore, the wavelength of N -photon cluster must be N times smaller than the wavelength λ of one photon:

$$\lambda_N = \frac{\lambda}{N}, \quad (1)$$

where λ_N is the wavelength of N -photon cluster. Reduction in the wavelength (1) when photons stick together was repeatedly confirmed in multiphoton experiments involving up to six coalescent photons [7–10]. Therefore, (1) may be considered a reliably established rule.

Phase velocity of a particle is determined by its frequency and wavelength $v_{ph} = \nu\lambda$. Photons do not interact, so when they stick together their energy $\varepsilon = h\nu$ does not change, hence, their frequency remains constant. So, when photons stick together, their phase velocity changes as the wavelength. Phase velocity of a single photon equals the velocity of light $v_{ph} = \nu\lambda = c$. Therefore, phase velocity of N -photon cluster, due to (1), is

$$v_{ph} = \nu\lambda_N = \frac{c}{N}, \quad (2)$$

which coincides with the speed of light only for single photons. Thus, phase velocity of N -photon cluster field in vacuum is N times less than the speed of light.

The Maxwell's equations for a plane wave in vacuum, however, cannot produce any phase velocity other than the speed of light. Therefore, the N -photon cluster field does not obey the Maxwell's equations.

In this regard, a natural question arises, what equations may describe a photon cluster field?

B. Search for suitable equations

Since a photon has spin $s = 1$, a two-photon cluster that consists of two indistinguishable photons has spin $s = 2$. It is a common belief that a spin-2 particle must be described by a second rank tensor field in contrast to a spin-1 particle that is described by a vector field. For this reason, a two-photon cluster must be described by a second rank tensor field. Maxwell's equations, of course, are only suitable for tensor fields of rank one. That is another reason why photon clusters do not obey Maxwell's equations.

It should be noted that L. Landau was the first to prove (in 1948) that two photons in a mode should be described by a second rank tensor field [11]. Such tensor field was then believed to obey some linear differential equations other than the Maxwell's equations.

A search for equations that could describe spin-2 mass-

less fields was conducted within the field-theoretic approach during almost the entire second half of the 20th century.

At the beginning, different linear equations were studied. Many options were considered but later a theorem was proved that any system of linear equations for a second rank tensor field would inevitably produce insurmountable contradictions if considered on the curved background, i. e. in the presence of external gravitational field [12]. Thus, no linear equations for spin-2 fields are possible.

Then various nonlinear equations for the tensor field were investigated. Several different systems of nonlinear equations were found that met all the requirements and did not lead to any contradictions. However, later it was proved in [13] and [14] that any such system of nonlinear equations could be reduced to the Einstein's equations of general relativity by appropriate coordinate transformations. Thus, all these nonlinear equations were actually the Einstein's equations in various, sometimes exotic, coordinate systems. A detailed discussion of this issue can be found in [15].

The main result of these studies may be given as follows: a consistent description of massless rank-2 tensor field is only possible within the Einstein's equations. Consequently, a two-photon cluster field must obey the Einstein's equations, as it is a spin-2 massless field.

C. Another approach

The same result can be obtained using a different method. There is a simple quantum principle, which states that if two quantum particles are *identical* then they must have the same quantum numbers. For example, two identical particles must have the same mass, charge, spin, etc. The reverse is also true: if two particles have the same quantum numbers then they are identical.

A two-photon cluster is a spin-2 massless particle. The same quantum numbers are known to characterize gravitons. Consequently, a two-photon cluster is identical to graviton and must, therefore, obey the same Einstein's equations that are believed to describe gravitons. This conclusion can be formulated in short: there is only one spin-2 massless particle in nature [14].

That means that in the 1987 Hong-Ou-Mandel experiment [2], *optical frequency gravitons* were registered at the output ports of the beam splitter. Such gravitons very much differ from those present in the Earth's gravitational field.

Indeed, experimentally observed gravitons are *real* particles while a stationary gravitational field contains *virtual* gravitons like electrostatic fields contain virtual photons. The difference between virtual and real particles is tremendous – a virtual particle cannot propagate to infinity while real particles may carry energy away to an infinite distance. In addition, virtual gravitons in the Earth's field have wavelengths that are determined by a typical distance over which the Earth's gravitational field extends, that is about tens of thousands of kilome-

ters. Gravitons in the HOM-type experiments have a wavelength less than a micron, i.e. at least 12 orders of magnitude smaller than wavelengths of virtual gravitons in the Earth's field. Finally, gravitons in the Earth's field are unpolarized particles, while in the experiments polarized gravitons are observed. These differences explain why gravitons observed in HOM-type experiments are so different from the virtual gravitons that make up the Earth's gravitational field.

Any deviation from an ideal alignment in a HOM-type experiment will result in a mixture of tensor and vector fields propagating in output modes c and d (Figure 1). In this case the efficiency of photon coalescence will be less than 100% and single photons will sometimes appear in two output modes. Probability of such event is defined by the vector field amplitude. That is the usual situation in such experiments.

If a two-photon cluster is identical to graviton then, within this paradigm, a 3-photon cluster should be a kind of coalescent state of photon and graviton, while a 4-photon cluster is the coalescent state of two gravitons. Since multiphoton clusters of various ranks have been observed in several experiments [7–10], it seems probable that there is a great variety of high-spin massless fields, representing different combinations of coalescent photons and gravitons. In the next Section, this conjecture will be supported by a sound validation within the framework of quantum statistics.

III. QUANTUM STATISTICS AND GRAVITY

The BE statistics is usually considered in a single cell of phase space (that corresponds to one coherence volume):

$$p_n(1) = \frac{w^n}{(1+w)^{n+1}}, \quad (3)$$

where $p_n(1)$ is the probability that n photons are in one coherence volume, w is the average number of photons per coherence volume.

For arbitrary phase-space volume τ , the BE statistics has the form:

$$p_n(\tau) = C_{\tau+n-1}^n \frac{w^n}{(1+w)^{n+\tau}}, \quad (4)$$

where $C_{\tau+n-1}^n$ is a binomial coefficient:

$$C_{\tau+n-1}^n = \frac{(\tau+n-1)!}{n!(\tau-1)!} = \frac{\tau(\tau+1)\dots(\tau+n-1)}{n!}. \quad (5)$$

Formula (4) was derived by Leonard Mandel for an integer number of cells [16]. It was later shown in [17] that the Mandel's formula (4) is valid for an arbitrary volume τ including nonintegral number of coherence volumes. It is clear that the last expression in (5) makes sense for any positive value of $\tau > 0$. If $\tau = 1$ then (4) becomes the usual expression (3) for the BE statistics in

a single cell.

Coalescence of particles in the BE statistics is a formal consequence of the fact that the BE statistics (4) coincides with a negative binomial distribution, which has the form

$$p_n(\tau) = C_{\tau+n-1}^n p^\tau (1-p)^n, \quad (6)$$

where coefficients $C_{\tau+n-1}^n$ are defined in (5), parameters p and τ must satisfy $0 \leq p \leq 1$ and $\tau > 0$, respectively.

If parameter

$$p = \frac{1}{1+w} \quad (7)$$

then (6) coincides with the BE statistics (4).

According to [3], probability distribution (6) is a special case of *Compound Poisson Distribution*, so that (6) describes a flow of random composite events. It was established in [3] that these composite events obey Poisson statistics

$$g_k(\tau) = \frac{(\eta\tau)^k}{k!} e^{-\eta\tau}, \quad (8)$$

where η is the average number of composite events per unit volume

$$\eta = \ln \frac{1}{p}, \quad (9)$$

while probability f_k that a composite event consists of k elementary events is given by a logarithmic distribution:

$$f_k = \frac{(1-p)^k}{k\eta}. \quad (10)$$

In the case of photon statistics, an elementary event may only be the registration of one photon by an ideal detector. Then a composite event will be a simultaneous registration of several photons, i.e. of photon cluster.

The above results, obtained in [3] from the general theory of Compound Poisson Distribution with respect to (6), were later derived in [6] from the BE statistics without resorting to Compound Poisson Distribution. Such a new derivation provides an independent confirmation of these results.

Due to (7), equations (9) and (10) take the form

$$\eta = \ln(1+w), \quad (11)$$

$$f_k = \frac{w^k}{k(1+w)^k \ln(1+w)}. \quad (12)$$

In the case of quantum statistics, (11) gives the average number of photon clusters per coherence volume (if no distinction is made between clusters of different ranks), while (12) is the probability that a photon cluster consists of k photons, $k = 1, 2, 3, \dots$

In the BE statistics, the average number of photons per

coherence volume is

$$w = \frac{1}{\exp(\beta\varepsilon) - 1}, \quad (13)$$

where $\beta = 1/kT$, and $\varepsilon = h\nu$. Therefore, distribution of photon clusters by rank (12), in view of (13), is a function of radiation frequency and blackbody temperature.

It follows from the above that quantum statistics predicts both the existence of photon clusters and distribution of clusters by rank starting from single photons, when $k = 1$, to infinite cluster ranks for $k \rightarrow \infty$. It follows from (12), however, that the relative frequency of occurrence of high-rank clusters rapidly falls with increasing the rank. In other words, single photons are most frequently met in the blackbody radiation while two-photon clusters are less frequent, etc. This issue was investigated in greater detail in [6] where spectra of cluster radiation in blackbody cavity were also found.

Thus, the mathematical fact that the BE statistics is actually a Compound Poisson Distribution implies that there are photon clusters of various ranks in blackbody radiation.

If a two-photon cluster is really identical to graviton, as it follows from Section II, then the BE statistics predicts that the blackbody radiation contains gravitons and other high-spin massless fields corresponding to various N -photon clusters.

IV. CONCLUSIONS

In this work it is shown that the Hong-Ou-Mandel effect is directly related to quantum gravity. First, it is proven that coalescent photons appearing in the HOM-effect do not obey the Maxwell's equations for electromagnetic field. Second, strong arguments are given in favor of conclusion that two coalescent photons (referred to as a two-photon cluster) must obey the Einstein's equations of general relativity. Therefore, a two-photon cluster is actually an optical frequency graviton. For this reason, the Hong-Ou-Mandel experiment [2] and many other HOM-type experiments in the field of quantum information are actually directly related to quantum gravity.

The idea behind this conclusion is simple: two identical photons, due to their bosonic nature, may occupy the same quantum state that, according to [11], must be described by a second-rank tensor field. Such tensor field, according to [13–15], can only obey the Einstein's equations of general relativity.

In the second part of this work it is shown that the existence of gravitons, as well as an infinite family of massless fields with higher spins, is an inevitable consequence of quantum statistics. The infinite hierarchy of high-spin massless fields appears as a set of composite events in the Compound Poisson Distribution that describes the Bose-Einstein statistics.

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